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LETTER TO THE EDITOR

Migdal–Kadanoff approach to superfluid film formation near a wall in ^{3}He–^{4}He mixtures

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Abstract. The semi-infinite generalised Blume-Emery-Griffiths model of ^{3}He-^{4}He mixtures with a vector order parameter is analysed by means of Migdal-Kadanoff recursion relations. Superfluid film formation near the wall is put into evidence as a Kosterlitz-Thouless type transition. At higher ^{3}He concentrations one observes the separation of a ^{4}He rich normal film near the wall.

Superfluid film formation near the wall in ^{3}He-^{4}He mixtures, first observed many years ago by Keyston and Laheurte (1967) (see also Gearhart and Zimmermann 1974, Romagnan *et al* 1978, Ruppeiner *et al* 1980) has been recently considered within the general theory of surface and special transitions (see e.g. Binder 1984) by Leibler and Peliti (1984). The physical mechanism leading to this phenomenon is quite well understood (see e.g. Laheurte *et al* 1977). The van der Waals interactions between the wall and ^{3}He or ^{4}He atoms are equal, but ^{3}He atoms occupy a larger volume because of their larger zero-point motion. One observes therefore a higher ^{4}He concentration near a wall, which may induce a local superfluid ordering.

This phenomenon was investigated by mean field techniques (Laheurte *et al* 1977, Leibler and Peliti 1984), and by real space renormalisation group, applied to a semi-infinite version of the model introduced by Blume *et al* (1971, BEG) to describe ^{3}He-^{4}He mixtures (Peliti and Leibler 1984). While the results were encouraging, as far as agreement of the calculated phase diagram with the experimental one was concerned, this approach was unsatisfactory, since the BEG model mimics superfluid ordering (the breaking of a *continuous* U(1) symmetry) via ferromagnetic ordering in a Ising-like system (the breaking of a *discrete* Z_2 symmetry). This is especially relevant for two-dimensional systems, like the film we are concerned with, because the Mermin-Wagner-Hohenberg theorem (see e.g. Hohenberg 1967) states that no spontaneous breaking of a continuous symmetry may take place in two dimensions. As a consequence, the film is superfluid since it is in the low-temperature phase of a Kosterlitz-Thouless (KT) transition (see e.g. Kosterlitz 1980), and not because its order parameter is different from zero. (Recent high-quality measurements (McQueeney *et al* 1984) have confirmed the apparentness of superfluid film formation, in a certain concentration range, to the KT universality class, putting into evidence the corresponding universal jump in superfluid density.) A similar problem arises in the description of two-dimensional ^{3}He-^{4}He mixtures, and was solved by the introduction, due to Berker and

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Nelson (1979) and to Cardy and Scalapino (1979), of the generalised BEG model. The order parameter of this model is a two-dimensional unit vector, mimicing the phase of the wavefunction of ^{4}He atoms. Two-dimensional versions of this model were investigated by the above authors by means of Migdal-Kadanoff recursion relations (Migdal 1975, Kadanoff 1976). These authors reached the conclusion that the tricritical point characteristic of the three-dimensional phase diagram was resolved, in two dimensions, into a *phase separation* critical point between two normal phases differing in He concentration, and a *critical end point* on the high He branch of the corresponding coexistence curve, at the end of the line of KT transition. If such a feature were preserved in the surface behaviour of the semi-infinite system, one would observe, in a certain concentration range, the separation of a surface normal film, rich in ^{4}He, followed at a lower temperature by a KT transition into a superfluid state.

We have checked this possibility by Migdal-Kadanoff analysis of the semi-infinite generalised BEG model. While the agreement in the location of the relevant known phase diagram features is not improved with respect to the simplified version (Peliti and Leibler 1984; this is mainly due to the poor quality of Migdal-Kadanoff recursion relations on the *three-dimensional* model) the main qualitative features of the phase diagram are confirmed. One observes indeed a line of superfluid film formation of the KT type branching from the λ line and extending in the higher ^{3}He concentration region. This line ends at a critical end point against a line of superfluid phase separation, which ends itself in a critical point. We show in figure 1 the experimental phase diagram of ^{3}He-^{4}He mixtures (from Romagnan *et al* 1978) with the experimental points which locate superfluid film formation, and in figure 2 the corresponding phase diagram obtained from our analysis.

The model is defined as follows. One considers a D-dimensional semi-infinite simple cubic lattice. To each point i are associated the variables $\sigma_i = 0, 1$ (0 represents

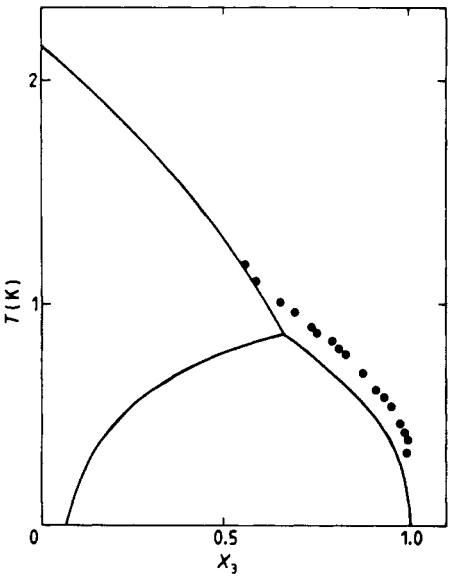

Figure 1. Phase diagram for ^{3}He-^{4}He mixtures, showing ^{3}He concentration on abscissae and temperature on ordinates. Experimental data for surface transitions, taken from Romagnan *et al* (1978), and due to several experimental groups.

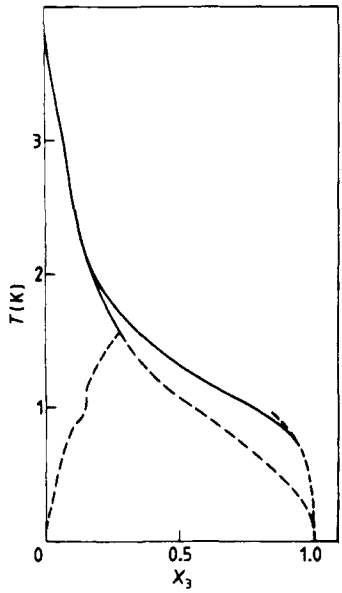

Figure 2. Phase diagram of the semi-infinite generalised BEG model from the Migdal-Kadanoff analysis. Temperature scale is fixed by the tricritical point temperature. Full curves correspond to continuous transitions, broken curves correspond to first-order transitions (bulk and surface).

^{3}He atoms; 1, ^{4}He atoms) and θ_i $(0 \leqslant \theta_i < 2\pi)$, which represents the phase of the wavefunction. The Hamiltonian is given by

$$-\beta H = \sum_{\langle ij \rangle} [\sigma_i \sigma_j V_{ij}(\theta_i - \theta_j) - K_{ij}(\sigma_i - \sigma_j)^2] + \sum_i \mu_i \sigma_i \tag{1}$$

where the first sum runs over all distinct nearest-neighbour pairs in the lattice, while the second runs over all points in the lattice. The first term represents the ordering interaction, if ^{4}He atoms are present at neighbouring lattice sites. The second one represents the preference for like atoms to occupy neighbouring positions. The function $V_{ij}(\theta)$ is periodic in its argument, with a period 2π. The interactions $V_{ij}(\theta)$ and K_{ij} are different if both i and j belong to the surface of the lattice or if one of them (or both) belong to the bulk. We have

$$V_{ij}(\theta), K_{ij} = \begin{cases} V(\theta), K & \text{if } i \text{ or } j \text{ (or both)} \in \text{Bulk} \\ V^s(\theta), K^s & \text{if } i \text{ and } j \in \text{Surface} \end{cases} \tag{2}$$

where

$$V(\theta) = J(\cos\theta - 1), \qquad V^s(\theta) = J^s(\cos\theta - 1). \tag{3}$$

We have likewise

$$\mu_i = \mu \quad \text{if } i \in \text{Bulk} \qquad \mu_i = \mu^s \quad \text{if } i \in \text{Surface}. \tag{4}$$

The difference between μ and μ^s represents the relative preference of ^{4}He atoms to be located near the surface.

Recursion relations for this model are obtained (Crisanti 1985) by the usual bond-moving and decimation approach (Kadanoff 1976). They describe the change

in interactions corresponding to a change $a \to ba$ in the lattice constant. A fraction $\alpha = K/(1+K)$ of the one-body potential for each lattice point is moved along with the bonds which end at that point (Cardy and Scalapino 1979). In this way the exact trivial behaviour both at $K \to 0$ ($T \to \infty$) and $K \to \infty$ ($T \to 0$) is recovered. Since the transformation does not conserve the form (3) of the V interactions, one has to consider the recursion relations for the Fourier components $e^{\tilde{V}(s)}$ of the periodic function $e^{V(\theta)}$. We only consider the first 40 coefficients in the Fourier expansion, but already from the fifteenth onwards they are 10^{-13} times the first ones, except at very low temperatures.

We consider the flow with a relatively small value of b ($=1.1$) to control erratic behaviour near $T=0$ fixed points. The flow is analysed to identify the phase one is in, and at the same time the bulk free energy is calculated by means of the method introduced by Nauenberg and Nienhuis (1974). The ^{4}He concentration is then obtained by numerical differentiation with respect to μ.

The identification of the KT transition requires some care. In an exact transformation one observes a line of fixed points in the flow of V^s which ends at the transition point. Due to the Migdal-Kadanoff approximation on the one hand, and the truncation of the Fourier expansion on the other, the line of fixed points appears with a very small but systematic drift towards higher temperatures. As a consequence, the transition is located with some arbitrariness at the point where this drift becomes important (José *et al* 1977, Berker and Nelson 1979, Cardy and Scalapino 1979). (No such problems appear in the location of the three-dimensional λ transition.) By defining the 'effective' temperature T_{eff} via

$$T_{\text{eff}}^{-1} = \sum_s s^2 e^{\tilde{V}(s)}, \tag{5}$$

we locate the transition for a two-dimensional system at $T_{\text{eff}} = 0.75$. If upon iteration, when the bulk parameters go to the trivial high-temperature fixed point, the surface T_{eff} goes towards a value lower (higher) than 0.75 we identify the surface as being in the low- (high-) temperature phase of a KT transition. The absence of a true fixed point makes difficult the study of the special point, where the KT surface line merges into the bulk λ line.

The starting point of the iteration equations is given by (3), with

$$J = Js = 1/T, \tag{6}$$

and

$$K = Ks = J/2 \qquad \text{(Blume-Capel limit)}. \tag{7}$$

One also sets

$$\mu^s = \mu + \phi/T \tag{8}$$

with $\phi = 2.37$, which is chosen in such a way that the temperature of the special transition is 1.37 times the tricritical temperature, as observed in the experiments (since the bulk phase diagram is reproduced rather poorly, we could not impose the He concentration to agree in the theoretical and experimental phase diagrams). By varying T and μ one obtains the diagram in figure 2.

We make the following observations on the phase diagram:

(i) in the limit of our precision, the surface transition line approaches the λ line at an angle, near the special transition point;

(ii) there is a small range of temperatures and concentration where one observes a surface *normal* fluid separation, followed by a KT transition; at higher ^{3}He concentrations, one observes a surface first-order phase separation into a superfluid state;

(iii) a small bulge inwards in the high ^{4}He concentration side of the bulk coexistence curve is due to an instability of the Blume-Capel limit and could be eliminated by a slight increase of K. It is not likely therefore to have direct physical significance.

We have shown in conclusion that a Migdal-Kadanoff analysis of the semi-infinite generalised Blume-Emery-Griffiths model reproduces the main features of the experimentally known phase diagram for superfluid film formation in ^{3}He-^{4}He mixtures. The possibility of phase separation of a normal film rich in ^{4}He should not be neglected in experimental investigations.

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References

Berker A N and Nelson D R 1979 *Phys. Rev.* B **19** 2488
Binder K 1984 in *Phase Transitions and Critical Phenomena* vol 8, ed C Domb and J L Lebowitz (London: Academic)
Blume M, Emery V J and Griffiths R B 1971 *Phys. Rev.* A **4** 1071
Cardy J L and Scalapino D J 1979 *Phys. Rev.* B **19** 1428
Crisanti A 1985 *Thesis* University of Rome
Gearhart C A and Zimmermann W 1974 *Phys. Lett.* **48A** 49
Hohenberg P C 1967 *Phys. Rev.* **158** 383
José J V, Kadanoff L P, Kirkpatrick S and Nelson D R 1977 *Phys. Rev.* B **16** 1217
Kadanoff L P 1976 *Ann. Phys., NY* **100** 359
Keyston J R G and Laheurte J P 1967 *Phys. Lett.* **24A** 132
Kosterlitz J M 1980 in *Phase Transitions in Surface Films* ed J D Dash (New York: Plenum)
Laheurte J P, Romagnan J P and Saam W F 1977 *Phys. Rev.* B **15** 4214
Leibler S and Peliti L 1984 *Phys. Rev.* B **29** 1253
McQueeney D, Agnolet G and Reppy J D 1984 *Phys. Rev. Lett.* **52** 1325
Migdal A A 1975 *Z. Eksp. Teor. Fiz.* **69** 1457 (1976 *Sov. Phys.-JETP* **42** 7433)
Nauenberg M and Nienhuis B 1974 *Phys. Rev. Lett.* **33** 1598
Peliti L and Leibler S 1984 *J. Physique Lett.* **45** L591
Romagnan J P, Laheurte J P, Noiray J C and Saam W F 1978 *J. Low Temp. Phys.* **30** 425
Ruppeiner G, Ryschkewitch M and Meyer H 1980 *J. Low Temp. Phys.* **41** 179